Electron shielding current in neutral beam current drive in general tokamak equilibria and arbitrary collisionality regime

Y. J. Hu,[1,2,a)] Y. M. Hu,[1,2] and Y. R. Lin-Liu[3]
[1]Institute of Plasma Physics, Chinese Academy of Sciences, Hefei, Anhui 230031, China
[2]Center for Magnetic Fusion Theory, Chinese Academy of Sciences, Hefei, Anhui 230031, China
[3]Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Taiwan

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A formula based on the solutions to the drift kinetic equation is proposed for modeling the trapped electron correction to the electron shielding current in neutral beam current drive in general tokamak equilibria and arbitrary collisionality regime. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.3693201]

In preparation for ITER operation, several neutral beam injection (NBI) and current drive (NBCD) models were reexamined and related computer codes benchmarked.[1–3] In this report, we revisit the kinetic theory[4–8] of the effect of trapped electrons on the electron shielding current in NBCD using Green's function formulation.[9,10] In particular, we extend the work of Ref. 7 to the finite collisionality regime. With some approximation, we propose that a formula for neoclassical bootstrap current given by Sauter *et al.*[11] be used to model the trapped electron effect in general tokamak equilibria and arbitrary collisionality regime.

In the presence of the fast ions generated by NBI, the perturbed electron distribution satisfies the following Fokker-Planck equation:

$$v_{\parallel}\hat{\mathbf{b}}\cdot\nabla f_{e1} - C_l(f_{e1}) = C^{e/f}(f_{em}), \tag{1}$$

where ∇ is the space gradient operator which is taken by holding the energy and magnetic moment constant, f_{em} and f_{e1} are electron equilibrium Maxwellian distribution and perturbed distribution function, respectively. $\hat{\mathbf{b}} = \mathbf{B}/B$, $\mathbf{B}$ is the equilibrium magnetic field, $v_{\parallel}$ is electron velocity parallel to the magnetic field, $C_l(f_{e1}) = C(f_{e1}, f_{em}) + C(f_{em}, f_{e1}) + C^{e/i}(f_{e1})$ is the linearized collision term including electron-electron and electron-ion collisions, $C^{e/i}(f_{e1})$ is the collision term of electrons with plasma ions, and $C^{e/f}(f_{em})$ is the collision term of electrons with fast ions, which is assumed to be known and acts as an inhomogeneous term in Eq. (1).

We want to determine the parallel (to the magnetic field) current density $j_{e\parallel}$ contributed by f_{e1}. It turns out that we can obtain $j_{e\parallel}$ through the following way. First, solve the adjoint equation

$$-v_{\parallel}\hat{\mathbf{b}}\cdot\nabla\chi_e - C_l(\chi_e) = q_e v_{\parallel} B f_{em} \tag{2}$$

to obtain the current response function χ_e, then $j_{e\parallel}$ can be expressed as

$$\langle j_{e\parallel}B\rangle = \left\langle \int d\mathbf{v}\frac{\chi_e}{f_{em}} C^{e/f}(f_{em})\right\rangle, \tag{3}$$

where q_e is the charge of electrons, $\langle\ldots\rangle$ is the flux-surface average, and $d\mathbf{v}$ is the volume element in velocity space. (The proof of Eq. (3) can be easily obtained by using the self-adjoint property of the operator $v_{\parallel}\hat{b}\cdot\nabla$ and C_l, i.e.,[9,10]

$$\left\langle \int d\mathbf{v} g v_{\parallel}\hat{\mathbf{b}}\cdot\nabla h\right\rangle = -\left\langle \int d\mathbf{v} h v_{\parallel}\hat{\mathbf{b}}\cdot\nabla g\right\rangle \tag{4}$$

and

$$\int d\mathbf{v} g C_l(f_{em}h) = \int d\mathbf{v} h C_l(f_{em}g), \tag{5}$$

where g and h are two arbitrary functions.)

In the usual situation of NBI, the fast ions beam velocity is much less than the electron thermal velocity, i.e., $v_f \ll v_{te}$. This should be applicable to ITER plasmas. In this case, the collision term of electrons with fast ions can be approximated as[5]

$$C^{e/f}(f_{em}) = \frac{m_e}{T_e}\nu_{ef} v_{\parallel} u_{f\parallel} f_{em}, \tag{6}$$

where $u_{f\parallel}$ is the average parallel velocity of fast ions. $\nu_{ef} = Z_f^2 n_f \nu_{ei}/(Z_{\rm eff} n_e)$, where n_e and n_f are the number density of electrons and fast ions, respectively, Z_f is the charge number of fast ions, and $Z_{\rm eff}$ is the effective charge number of plasma ions. $\nu_{ei} = \Gamma^{e/e} Z_{\rm eff}/v^3$ is the pitch angle scattering rate. $\Gamma^{e/e} = n_e e^4 \ln\Lambda^{e/e}/(4\pi\epsilon_0^2 m_e^2)$, where $\ln\Lambda^{e/e}$ is the Coulomb logarithm, $-e$, m_e, and T_e are, respectively, the charge, mass, and temperature of electrons, and ϵ_0 is the dielectric constant of free space. Using Eq. (6) in Eq. (3), we obtain

$$\langle j_{e\parallel}B\rangle = -\frac{Z_f}{Z_{\rm eff}}\frac{1}{Ip_e}\left\langle j_{f\parallel}B\int d\mathbf{v}\chi_e\nu_{ei}\frac{Iv_{\parallel}}{\Omega_e}\right\rangle, \tag{7}$$

where $j_{f\parallel} = Z_f e n_f u_{f\parallel}$ is the fast ion parallel current, $p_e = n_e T_e$, $\Omega_e = -Be/m_e$, $I = B_\phi R$ is a flux-surface function, B_ϕ is the toroidal component of the equilibrium magnetic field, and R is the usual cylindrical coordinate of points on the flux-surface.

To evaluate $\langle j_{e\parallel}B\rangle$ according to Eq. (7), one needs to know the response function χ_e and also the poloidal angular

a)Electronic mail: yjhu@ipp.ac.cn.

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dependence of $j_{f\|}$. In both banana and Pfirsch-Schlüter regimes, the term $D \equiv \int d\mathbf{v}\chi_e \nu_{ei} I v_\| /\Omega_e$ appearing in Eq. (7) can be shown to be a flux surface function. Thus, in these cases, D can be taken out of the flux surface average operator. In the case of the intermediate collisionality regime, D is not exactly a flux surface function and has poloidal angle dependent collisionality corrections. However, from the numerical results of neoclassical bootstrap current theory of Sauter *et al.*,[11] one might expect that these finite collisionality corrections in D are weakly dependent on the poloidal angle. Working from their numerical results for the dimensionless electron density gradient bootstrap current coefficients $\mathcal{L}_{31}$ and the electron screening factor in the ion temperature gradient coefficient $\mathcal{L}_{34}$, one can show that $\langle B^2 D\rangle \approx \langle B^2\rangle\langle D\rangle$ in a wide range of the collisionality parameter $\nu_{e\star}$, for general tokamak geometry. In particular, the difference between $\langle DB^2\rangle$ and $\langle D\rangle\langle B^2\rangle$ appears to be proportional to $\nu_{e\star}$, instead of $\sqrt{\nu_{e\star}}$, as $\nu_{e\star} \to 0$. Also, at $\nu_{e\star} = 1$ and the effective trapped particle fraction $f_t = 0.65$, which roughly corresponds to the inverse aspect ratio of 0.2, the difference between $\langle DB^2\rangle$ and $\langle D\rangle\langle B^2\rangle$ is always less than 5% for $Z_{\rm eff} = 1-5$. As for the poloidal angular dependence of $j_{f\|}$, due to large radial excursion of the fast ions, the poloidal angular dependence of $j_{f\|}$ deviates from the usual $j_{f\|} \propto B$ dependence in the thin banana width limit. On the other hand, suppose that $j_{f\|}$ could be modeled by a one-parameter function such as $\alpha B + (1-\alpha)\langle B^2\rangle/B$ with $0 < \alpha < 1$, which covers a reasonably wide range of the poloidal angular dependence, then the assumption that D be a flux-surface function will make a small error in predicting $\langle j_{e\|}B\rangle$. Based on the above arguments, we write Eq. (7) approximately, by taking D out of the flux surface average operator and replacing it by $\langle D\rangle$, as

$$\langle j_{e\|}B\rangle = -\langle j_{f\|}B\rangle \frac{Z_f}{Z_{\rm eff}}\frac{1}{Ip_e}\left\langle \int d\mathbf{v}\chi_e \nu_{ei}\frac{I v_\|}{\Omega_e}\right\rangle. \tag{8}$$

According to the neoclassical bootstrap current theory of Sauter *et al.*,[11] we have

$$\frac{1}{Ip_e}\left\langle \int d\mathbf{v}\chi_e \nu_{ei}\frac{I v_\|}{\Omega_e}\right\rangle = 1 - \mathcal{L}_{31}, \tag{9}$$

where $\mathcal{L}_{31}$ is the dimensionless electron density gradient bootstrap current coefficient. Thus, Eq. (8) is written as

$$\langle j_{e\|}B\rangle = -\langle j_{f\|}B\rangle \frac{Z_f}{Z_{\rm eff}}(1-\mathcal{L}_{31}). \tag{10}$$

The total current is the sum of the beam current carried by the fast ions and the electron shielding current, i.e., $j_\| = j_{f\|} + j_{e\|}$. Then, we have

$$\langle j_\| B\rangle = \langle j_{f\|}B\rangle\left[1 - \frac{Z_f}{Z_{\rm eff}}(1-\mathcal{L}_{31})\right], \tag{11}$$

and the ratio of the total current to the fast ion current

$$F \equiv \frac{\langle j_\| B\rangle}{\langle j_{f\|}B\rangle} = 1 - \frac{Z_f}{Z_{\rm eff}}(1-\mathcal{L}_{31}). \tag{12}$$

The formula of $\mathcal{L}_{31}$ given by Sauter *et al.*[11] is

$$\mathcal{L}_{31} = \left(1+\frac{1.4}{Z_{\rm eff}+1}\right)X - \frac{1.9}{Z_{\rm eff}+1}X^2 + \frac{0.3}{Z_{\rm eff}+1}X^3 + \frac{0.2}{Z_{\rm eff}+1}X^4, \tag{13}$$

with

$$X = \frac{f_t}{1+(1-0.1f_t)\sqrt{\nu_{e\star}} + 0.5(1-f_t)\nu_{e\star}/Z_{\rm eff}}, \tag{14}$$

where $\nu_{e\star}$ is the ratio of the electron collision frequency to the bounce frequency $\nu_{e\star} = qR\nu_{ee}/(\epsilon^{3/2}v_{te})$, q and ϵ are the safety factor and inverse aspect ratio of a toroidal magnetic surface, respectively, $v_{te} = \sqrt{T_e/m_e}, \nu_{ee}$ is the electron-electron collision frequency, $\nu_{ee} = \sqrt{2}n_e e^4 \ln\Lambda_e/(12\pi^{3/2}\epsilon_0^2\sqrt{m_e}T_e^{3/2})$, and f_t is the effective trapped fraction[12]

$$f_t = 1 - \frac{3}{4}\left\langle\frac{B^2}{B_{\max}^2}\right\rangle\int_0^1 \frac{\lambda d\lambda}{\left\langle\sqrt{1-\lambda B/B_{\max}}\right\rangle}. \tag{15}$$

We note that the formulas given by Eqs. (13)–(15) are valid for general tokamak equilibria and arbitrary collisionality regime. Thus, using these formulas in Eq. (10), we obtain a formula for the electron shielding current which should be applicable to all collisionality regimes in tokamak plasmas.

Previous models for the electron shielding current commonly used in various transport codes are well summarized in Refs. 1 and 2. In order to make a comparison with those models, we take the concentric circular cross-section equilibrium given in Ref. 7 and evaluate the ratio F as a function of the inverse aspect ratio ϵ and the electron collisionality parameter $\nu_{e\star}$ for the effective charge number of plasma ions $Z_{\rm eff} = 1.6$ and the fast ion charge number $Z_f = 1$. The results are plotted in Fig. 1. The small circles in the figure correspond to the results for the banana regime given in Refs. 7 and 8. We note that our results for F in the banana regime agree well with those obtained by Kim, Callen, and Hamnén[8] using the Hirshman-Sigmar moment approach[13] and those given in Ref. 7 which adopts a bootstrap current formula in

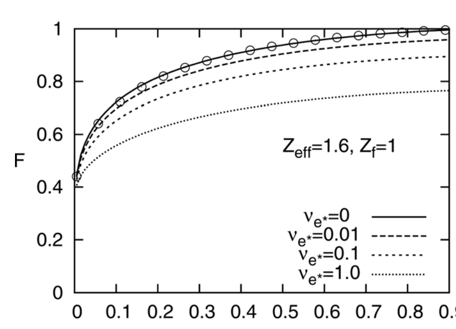

FIG. 1. The ratio F of the total neutral beam driven current to the fast ion current, Eqs. (12)–(15), as a function of the inverse aspect ratio ϵ in concentric circular flux-surface equilibrium. The different lines in the figure correspond to different values of the electron collisionality parameter, $\nu_{e\star} = 0$, 0.01, 0.1, and 1.0. The small circles in the figure are the results for the banana regime given in Refs. 7 and 8. $Z_{\rm eff} = 1.6$ and $Z_f = 1$.

the banana regime for general tokamak geometry given by Hirshman.[14] As is shown in the figure, the results for small values of $\nu_{e\star}$ are close to those in the banana regime as expected. The results in the figure also indicate that the ratio of the total current to the fast ion current is reduced when the value of $\nu_{e\star}$ is increased. This means the electron shielding current is actually increased when $\nu_{e\star}$ is increased. The explanation for this is that the collision increases the fraction of circulating electrons so that more electrons can contribute to the shielding current.

Up to now, there is only one collisional model for the electron shielding current in NBCD available in the literature, which is based on a formula given by Hirshman.[13,15,16] This model has been implemented in two widely used transport codes, TRANSP and ONETWO,[17,18] to model NBCD. While the formulation and final expressions of the model can be found in a number of paper,[13,15,16] a nice summary of the model and its code implementation in TRANSP are given in Ref. 2. (Hereafter, we will refer to it as the Hirshman1978 model.) Fig. 2 compares the ratio F calculated by the Hirshman1978 model with the one by Eqs. (12)–(15) in this report. Hirshman's formula is obtained in the case of small inverse aspect ratio and is valid only for $0.01 \leq \epsilon \leq 0.15$ (Ref. 2); thus, the comparison in Fig. 2 is limited to this range. The solid lines are results of the present work adopting Sauter's formulas for the bootstrap current coefficient, Eqs. (12)–(15), while the dashed lines are those of Hirshman's formula. The two sets of the curves show similar qualitative behaviors of F as a function of the inverse aspect ratio ϵ and the electron collisionality parameter $\nu_{e\star}$. The observed discrepancies are within 15%. The discrepancies can be attributed to several factors. The results of the Hirshman1978 model are obtained in the case of small inverse aspect ratio. In Fig. 2, one can see the trend that as $\epsilon \to 0$ the two sets of curves agree with each other better and better. For finite aspect ratio, the Hirshman results gradually deviate from those of the present work. This can be attributed to some finite aspect ratio corrections neglected in his model. Another source of the discrepancies could come from the difference in the interpolation schemes for deriving the final analytic expressions of the two models. Sauter's formulas, which we use here, are a direct fit of the code results of the bootstrap current coefficient based on the drift kinetic equation with the full electron-electron collision operator in general tokamak geometry. Hirshman's formulas are obtained based on the moment approach with an interpolation formula of viscosity moments from low and high collisionality regimes. Those moments are calculated with a reduced collision operator at the small inverse aspect limit.

In summary, we have showed that, for arbitrary aspect ratio and arbitrary collisionality regime, the electron shielding current in neutral beam current drive can be approximately expressed in terms of the electron density gradient coefficient

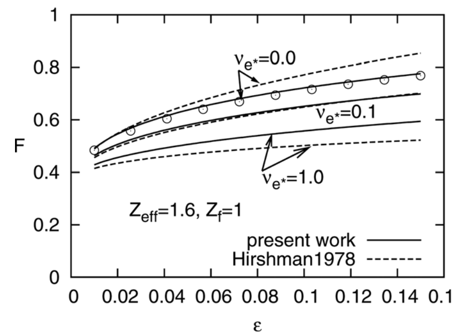

FIG. 2. Comparison of the predictions for the F factor from the present work and the Hirshman1978 model (Ref. 15). The solid lines are the results of the present work adopting Sauter's formulas, Eqs. (12)–(15), and the dashed lines are those of the Hirshman1978 model. The small circles in the figure are the results for the banana regime given in Refs. 7 and 8. $Z_{eff} = 1.6$ and $Z_f = 1$.

of the bootstrap current, $\mathcal{L}_{31}$. Thus, the existing formula for $\mathcal{L}_{31}$ valid for general tokamak equilibria and arbitrary collisionality regime provides a general formula for calculating the electron shielding current. These formulas [Eqs. (10)–(15)] for the electron shielding current can be easily included in transport codes to model neutral beam current drive.

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